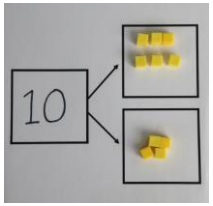
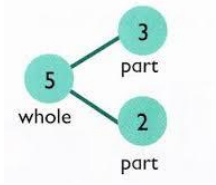
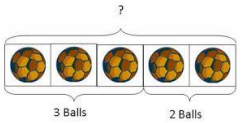
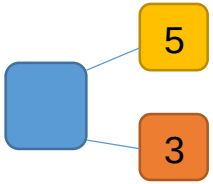
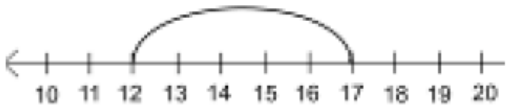
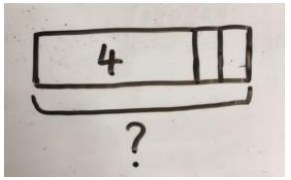
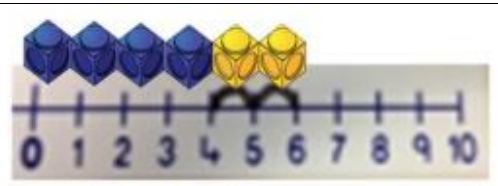
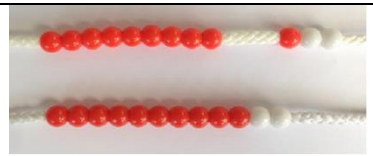
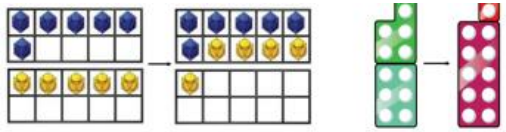
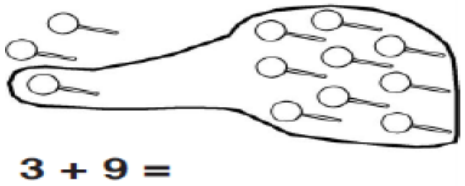
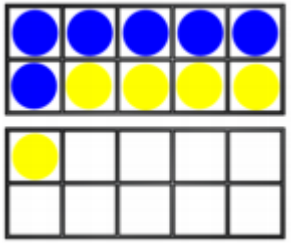
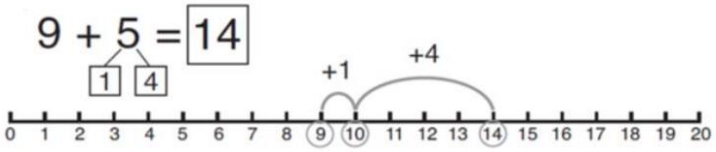
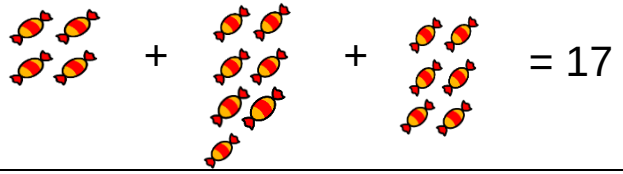
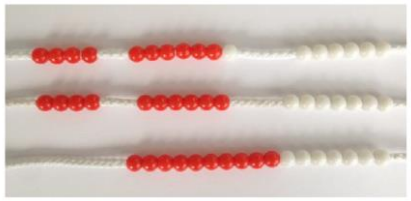
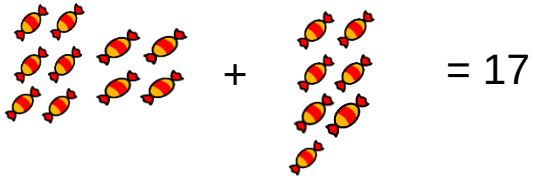
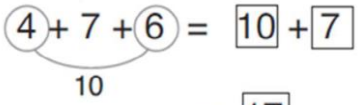
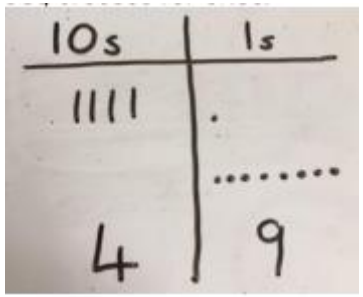
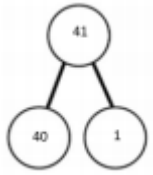
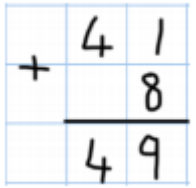
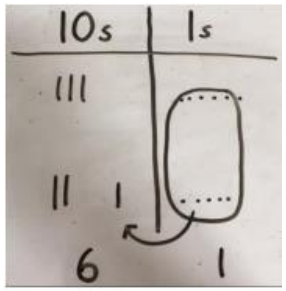
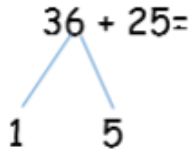
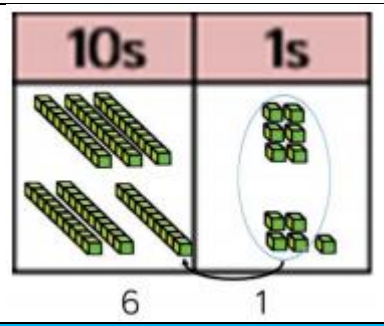
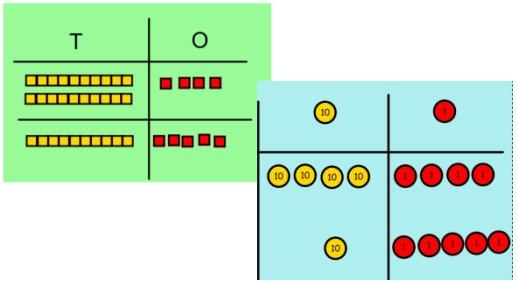
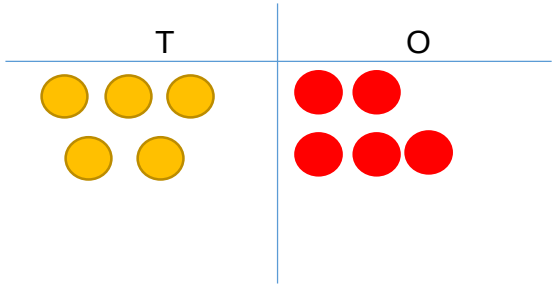
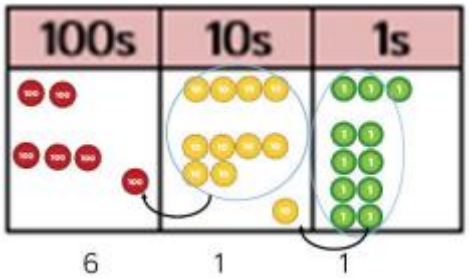
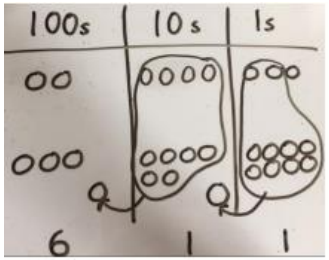
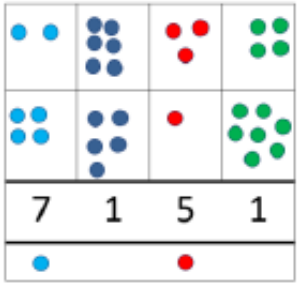
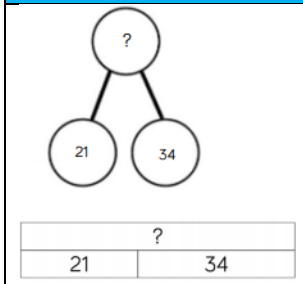
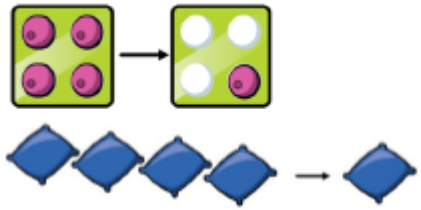
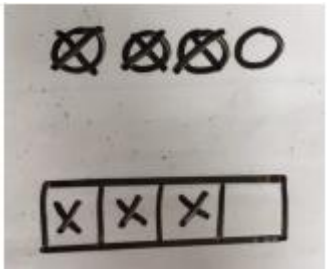
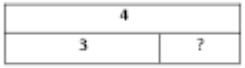
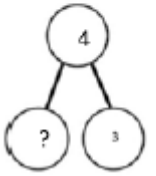
Addition Key language: sum, total, parts and wholes, plus, add, altogether, more, ‘is equal to’ ‘is the same as’			
Objective and Strategies	**Concrete**	**Pictorial**	**Abstract**
Combining two parts to make a whole: part-whole model	   Use cubes to add two numbers together as a group or in a bar. 	   Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Counting on using number lines (Using manipulatives eg. cubes or numicon)	 Initially start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	4 + 2  12 + 5 = 17 	What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2?  If children are ready use bar model 5 + 12 = 17

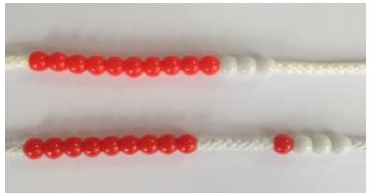
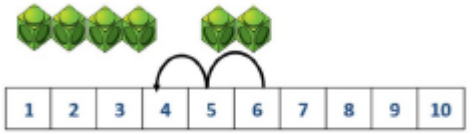
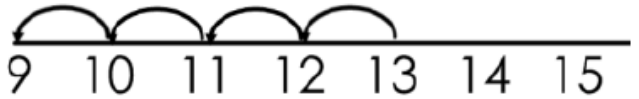
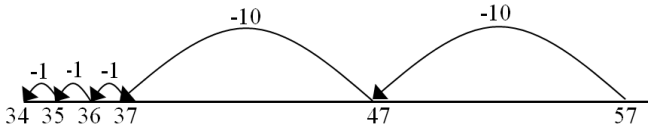
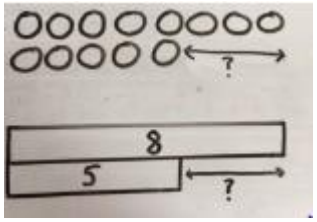
		Start at the larger number on the number line and count on in ones (or bigger jumps) or in one jump to find the answer if ready	Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10 (Using ten frames, counters/ cubes or numicon)	 6 + 5 = 11  Initially start with the bigger number and use the smaller number to make 10.	Use pictures or a number line. Regroup or partition the smaller number to make 10.  Children can draw their own ten frame with examples  	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now? 11= 6 + □ 6 + □ = 11 6 + 5 = 5 + □ 6 + 5 = □ + 4
Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.		

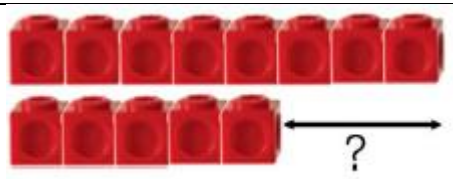
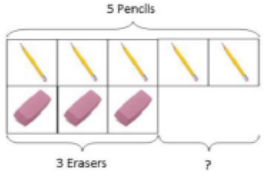
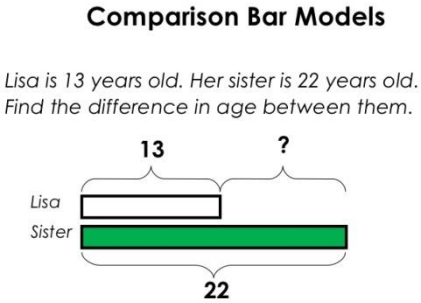
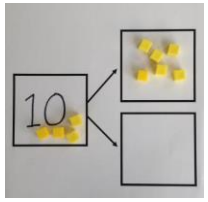
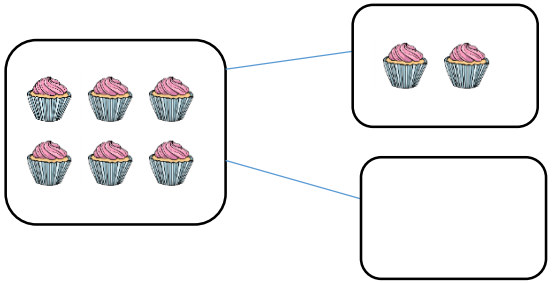
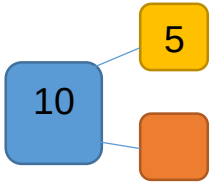
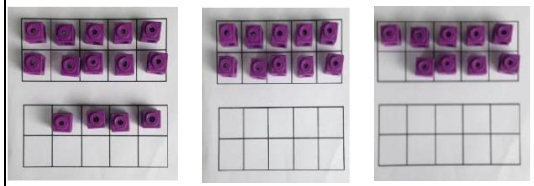
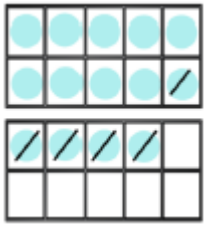
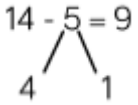
	 Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	Add together three groups of objects. Draw a picture to recombine the groups to make 10. 	 = 17 Combine the two numbers that make 10 and then add on the remainder.
TO + O using base 10.	Continue to develop understanding of partitioning and place value. 41 + 8 	Children to represent the base 10 e.g. lines for tens and dot/crosses for ones. 	41 + 8  1 + 8 = 9 40 + 9 = 49 
TO + TO using base 10.	Continue to develop understanding of partitioning and place value. 36 + 25	Children to represent the base 10 in a place value chart. 	Looking for ways to make 10. 30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 

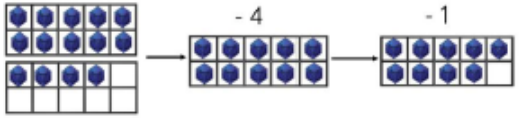
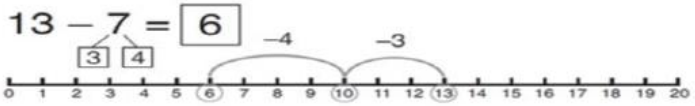
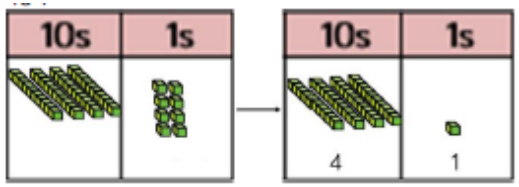
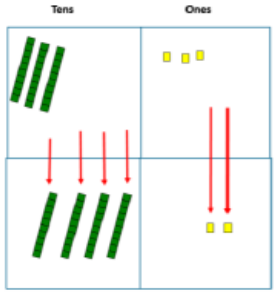
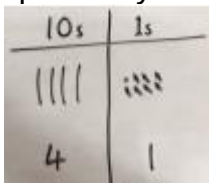
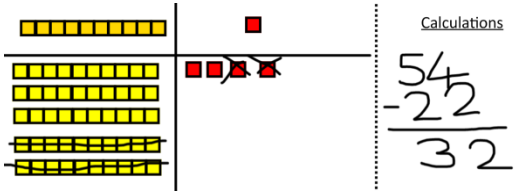
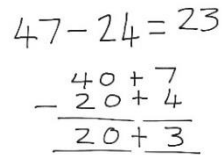
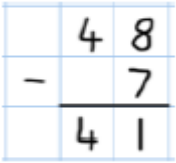
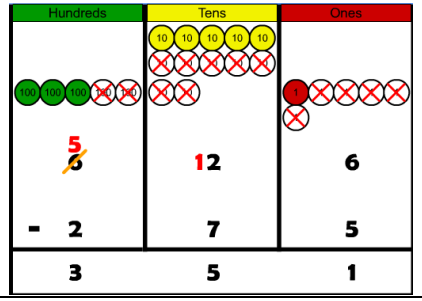
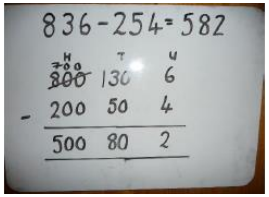
			Formal method 36 +25 61 1
EXPECTED KEY STAGE 2 STARTING POINT			
Column method- no regrouping	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters. 	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions. 	Calculations 21 + 42 = 21 + 42

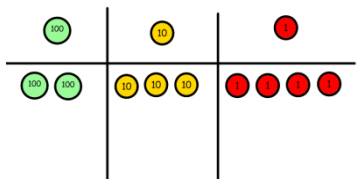
Column method- regrouping	Make both numbers on a place value grid. 243+ 368 = 611  Add up the units and exchange 10 ones for one 10. Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.  2634 + 4517 = 7151  	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 20 + 5 40 + 8 60 + 13 = 73 Move to formal written method 243 +368 611 1 1 As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. 72.8 + 54.6 127.4 1 1 £23.59 + £7.55 £31.14 1 1 1 93.511 2 1 2

Conceptual variation; different ways to ask children to solve 21 + 34			
	Word problems: In year 3, there are 21 children and in year 4, there are 34 children. How many children in total? 21 + 34 = 55. Prove it	21 +34 ____ **21 + 34 =** ☐ = 21 + 34 Calculate the sum of twenty-one and thirty-four.	 Missing digit problems: 
Subtraction **Key language: take away, less than, the difference, subtract, minus, fewer, decrease.**			
Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away from a whole. 4 – 3 = 1 	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. 	4- 3 = ☐ = 4 – 3  

Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count  backwards in ones. 13 – 4 Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track 6 - 2 = 4   Start at the bigger number and count back the smaller number showing the jumps on the empty number line.  	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.
Find the difference	Compare amounts and objects to find the difference. Use cubes to build towers or make bars to find the difference Calculate the difference between 8 and 5.	Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.  	Find the difference between 8 and 8 – 5, the difference is … Children to explore why these calculations 9 - 6 = 8 – 5 = 7 – 4 have the same difference.

	 Use basic bar models with physical objects to find the difference  	Draw bars to find the difference between two numbers.  	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.
Part Part Whole Model	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.
Make 10	14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you	Children to present the ten frame pictorially and discuss what they did to make 10. 	Children to show how they can make 10 by partitioning the subtrahend.  16 – 8=

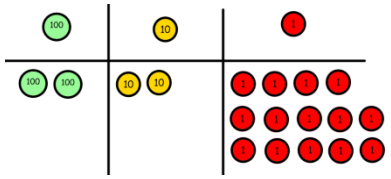
	have taken away 5. You are left with the answer of 9. 	Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer. 	How many do we take off to reach the next 10? How many do we have left to take off?
EXPECTED KEY STAGE 2 STARTING POINT			
Column method using base 10 without regrouping	Column method without regrouping 48 -7  	Children to represent the base 10 pictorially.  Draw the Base 10 or place value counters alongside the written calculation to help to show working. 	Use expanded method to consolidate  Column method or children could count back 7. 
Column method with regrouping	Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters		



Calculations

234
- 88

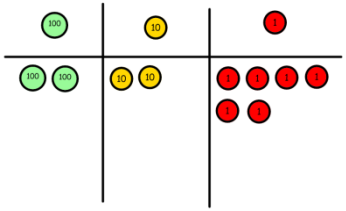
Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



Calculations

234
- 88

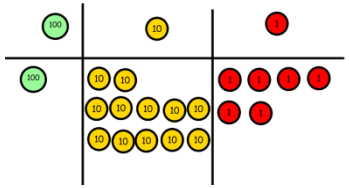
Now I can subtract my ones.



Calculations

234
- 88

Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



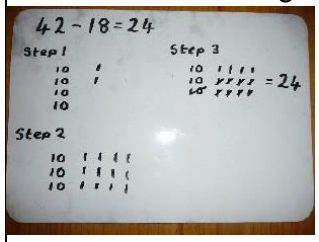
Calculations

234
- 88

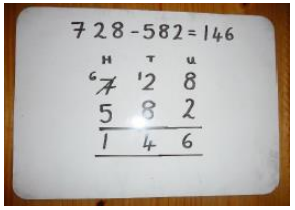
Now I can take away eight tens and complete my subtraction

Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

When confident, children can find their own way to record the exchange/regrouping.



Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.

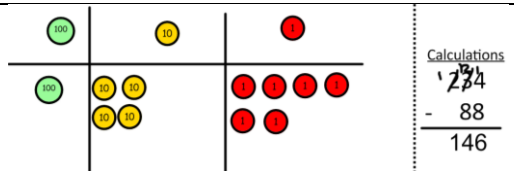
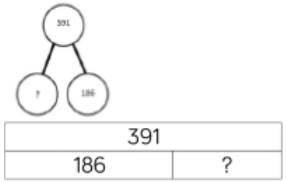
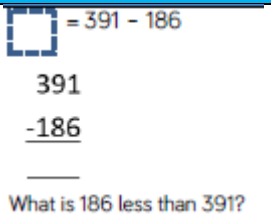
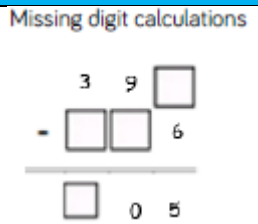


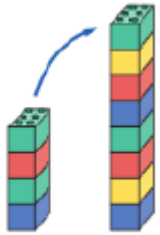
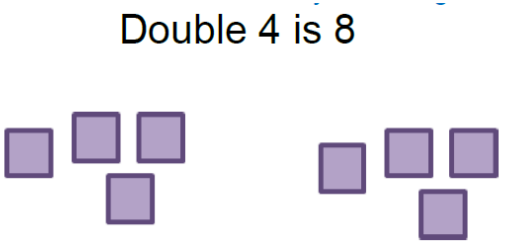
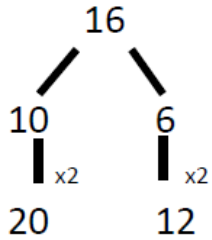
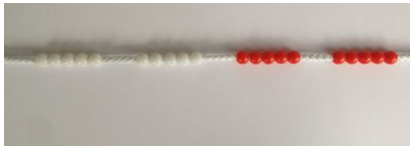
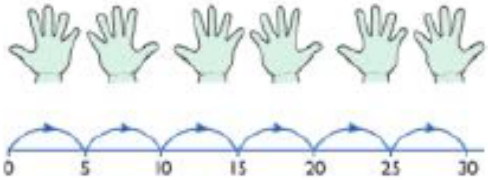
Children can start their formal written method by partitioning the number into clear place value columns.

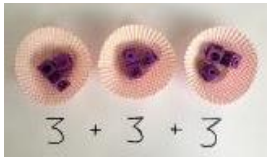
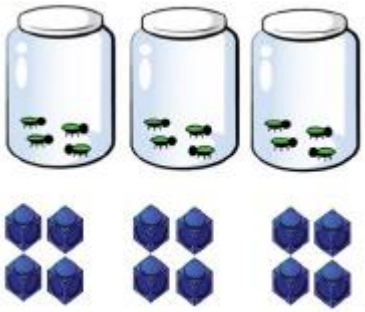
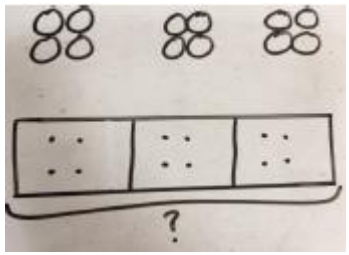
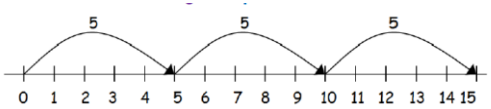
Moving forward the children use a more compact method.

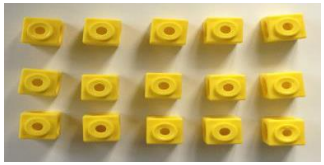
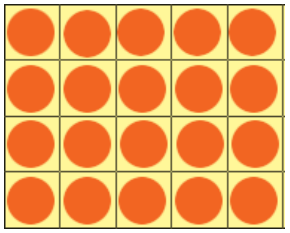
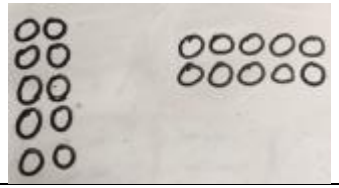
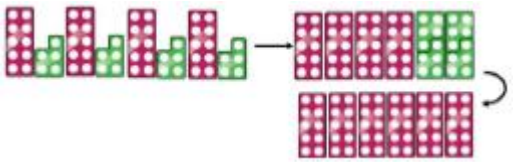
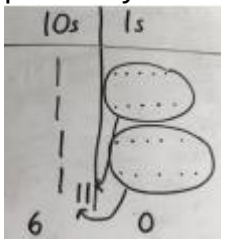
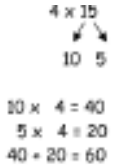
This will lead to an understanding of subtracting any number including decimals.

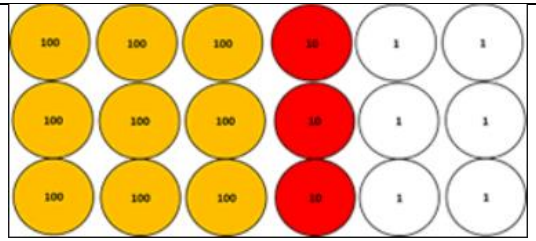
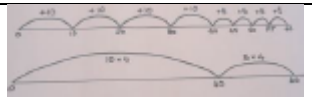
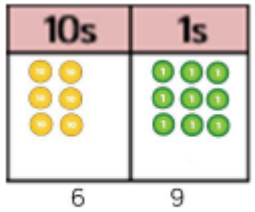
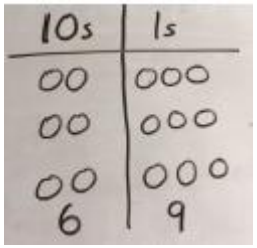
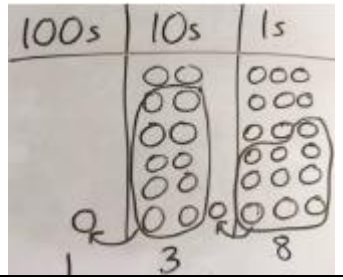
$$\begin{array}{rr} & {\scriptstyle 5\ \ 12\ \ \ \ 1} \\ & 2\ \not{6}\ \not{3}\ .\ \mathbf{0} \\ - & \ \ \ 2\ 6\ .\ 5 \\ \hline & 2\ 3\ 6\ .\ 5 \end{array}$$

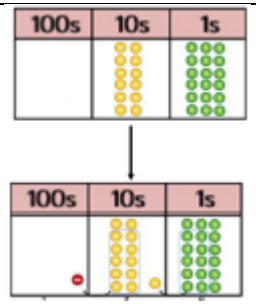
	 Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		
Conceptual variation; different ways to ask children to solve 391 - 186			
	Raj spent £391, Timmy spent £186. How much more did Raj spend? Calculate the difference between 391 and 186.		

Multiplication Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.			
Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.  double 4 is 8 4 × 2 = 8	Draw pictures to show how to double a number.  Double 4 is 8	 Partition a number and then double each part before recombining it back together.
Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

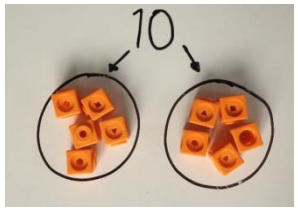
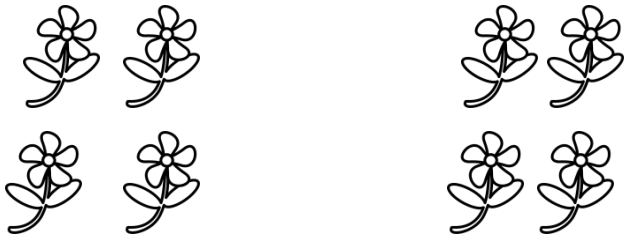
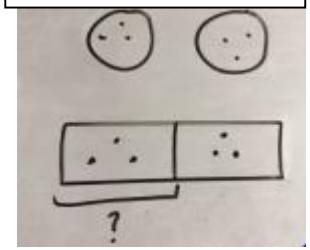
Repeated addition	Repeated grouping/repeated addition   Use different objects to add equal groups.  3 × 4 4 + 4 + 4 There are 3 equal groups, with 4 in each group. 	Children to represent the practical resources in a picture and use a bar model.  There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15	3 × 4 = 12 4 + 4 + 4 = 12 Write addition sentences to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10

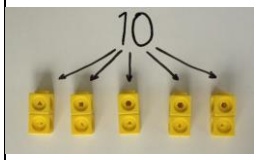
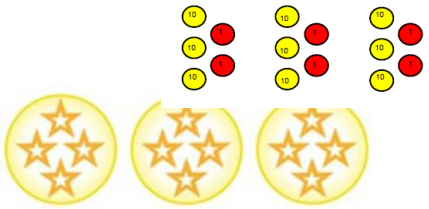
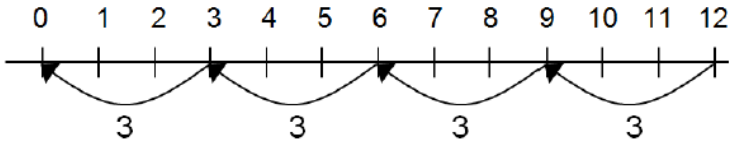
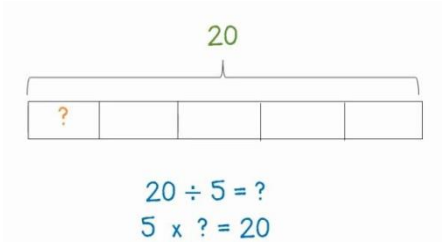
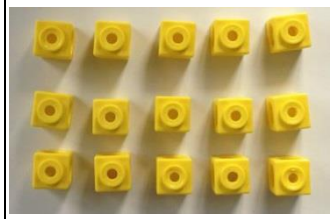
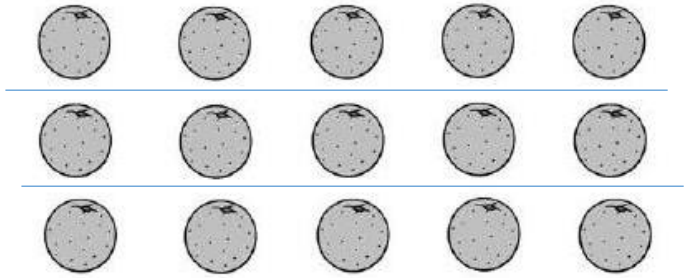
EXPECTED KEY STAGE 2 STARTING POINT			
Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.  4 × 2 = 8 2 × 4 = 8  2 × 4 = 8 4 × 2 = 8  Link arrays to area of rectangles. 	Use an array to write multiplication sentences and reinforce repeated addition. 3 x 5 =15 15 = 3 x 5 5 x 3 = 15 5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15
Partition to multiply using Numicon, base 10, place value counters or Cuisenaire rods.	4 x 15 =  312 x 3 =	Children to represent the concrete manipulatives pictorially.  10s 1s 6 0	Children to be encouraged to show the steps they have taken.  4 x 15 10 5 10 x 4 = 40 5 x 4 = 20 40 + 20 = 60 A number line can also be used

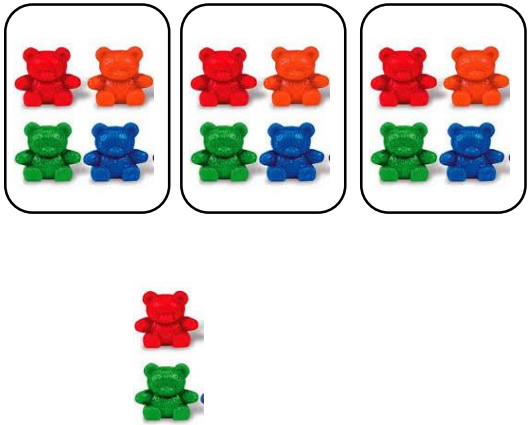
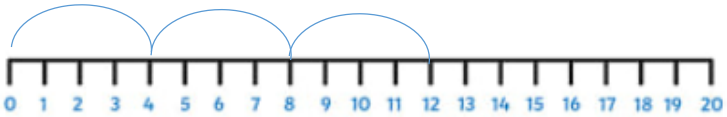
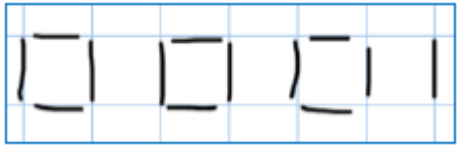
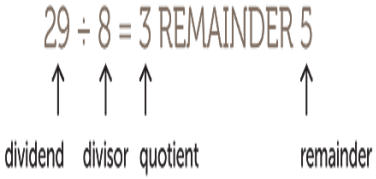
			
Column multiplication 2 X1 digit 3 x 1 digit	Children can continue to be supported by place value counters at the stage of multiplication. 3 x 23 =  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below. Formal column method with place value counters. 6 x 23	Children to represent the counters pictorially.  Children to represent the counters/base 10, pictorially e.g. the image below.  	23 × 3 69 When multiplying by 2 digit numbers start with expanded multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer.

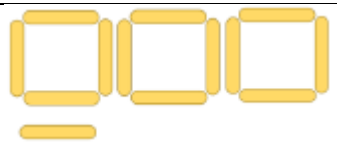
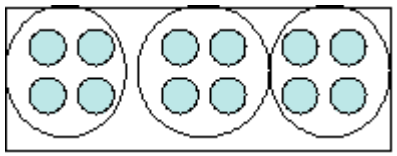
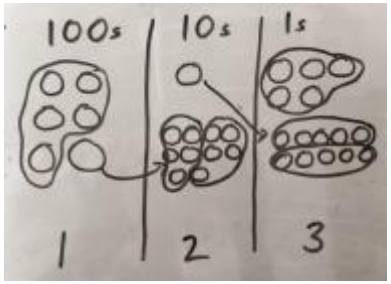
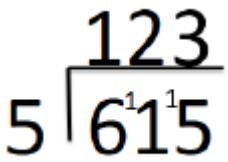
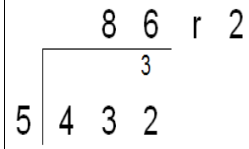
			32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768 Label columns if needed. H T O 4 7 6 X 4 1 9 0 4 3 2 Numbers are carried underneath. Carry is circled once it has been added on.
Column multiplication Long multiplication	Continue to support children with these methods if needed.	Continue to support children with these methods if needed.	Leave a line before writing the second number. Carries for the multiplication of the ones are carried above the calculation. Carries for the tens are then recorded in the space between the numbers.

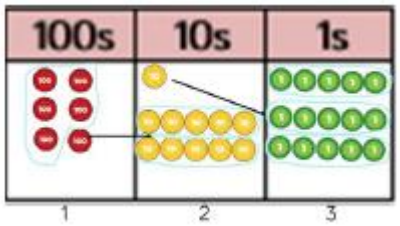
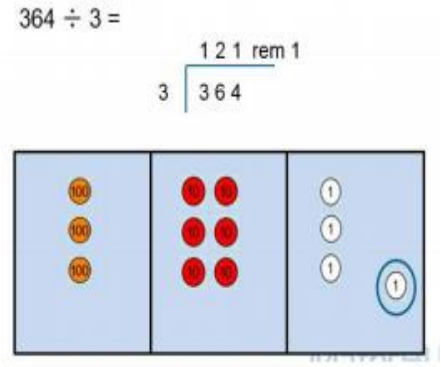
			All carried are to be circled as they are added on. 1 1 4 6 2 X 1 1 3 3 1 3 8 6 4 3 8 6 0 + 4 5 2 4 6 1 1 Carries are to be circled.
Conceptual variation; different ways to ask children to solve 6 × 23			
	Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week? With the counters, prove that 6 x 23 = 138	Find the product of 6 and 23 6 × 23 = □ = 6 × 23 6 × 23 23 × 6	What is the calculation? What is the product? 

Division Key language: share, group, divide, divided by, half.			
Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	 I have 10 cubes, can you share them equally in 2 groups? 	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4 6 ÷ 2 = 3 	Share 9 buns between three people. 9 ÷ 3 = 3 6 ÷ 2 = 3 3 | 3

Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.   96 ÷ 3 = 32 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?
EXPECTED KEY STAGE 2 STARTING POINT			
Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created.	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7

	Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15		
Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over  2d ÷ 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used. 13 ÷ 4 Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. 13 ÷ 4 – 3 remainder 1  Draw dots and group them to divide an amount and clearly show a remainder.  Children to represent the lollipop sticks pictorially. 	13 ÷ 4 – 3 remainder 1 Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line. ‘3 groups of 4, with 1 left over’  Complete written divisions and show the remainder using r. 

	 There are 3 whole squares, with 1 left over.	There are 3 whole squares, with 1 left over.	
Short division	Short division using place value counters to group. 615 ÷ 5 1. Make 615 with place value counters. 2. How many groups of 5 hundreds can you make with 6 hundred counters? 3. Exchange 1 hundred for 10 tens. 4. How many groups of 5 tens can you make with 11 ten counters? 5. Exchange 1 ten for 10 ones. 6. How many groups of 5 ones can you make with 15 ones?	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.  Encourage pupils to move towards counting in multiples to divide more efficiently.	Children to the calculation using the short division scaffold.  **BY THE END OF YEAR 4** Move onto divisions with a remainder. 

Progress to long division YEAR 6	  	Write down higher multiplication tables to help with trickier numbers: 16 32 48 64 80 96 112 128	**YEAR 5 AND 6** Divisions with a remainder 8 6 r 2 3 5 | 4 3 2 **Once confident, express the remainder as a fraction or decimal Then:** 1 8 6 1/5 5 | 9 ⁴3 ³1 1 4 . 6 16 21 3 5 | 5 1 1 . 0 Children to the calculation using long division with times tables fact to support.

			13 15 1989 15 48 45 39 1 x 15 =15 2 x 15 =30 3 x 15 =45
Conceptual variation; different ways to ask children to solve 615 ÷ 5			
Using the part whole model below, how can you divide 615 by 5 without using short division?  	I have £615 and share it equally between 5 bank accounts. How much will be in each account? 615 pupils need to be put into 5 groups. How many will be in each group?	5 ⟌ 615 615 ÷ 5 = □ = 615 ÷ 5	What is the calculation? What is the answer?